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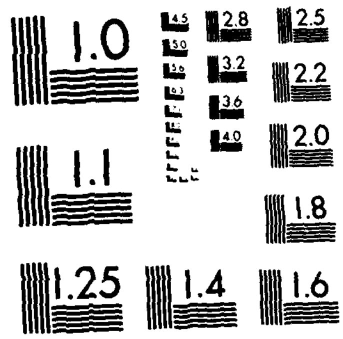
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THE PRICE AND PRODUCTION LEVEL
OF THE DETERIORATING
INVENTORY SYSTEM

Dae Won Kim, Captain, Korea Army

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This paper presents a modified model for a deteriorating inventory system determining price and production levels. Specifically, the exponential distribution is used to represent the distribution of the time to deterioration. The optimal production lot size is derived under conditions of continuous review, constant demand and no shortages. The sensitivity to changes in perishability and product price is considered. Finally, a numerical example is solved to show the impact of price and deterioration and to derive the optimal production lot size.

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THE PRICE AND PRODUCTION LEVEL OF THE DETERIORATING INVENTORY SYSTEM

A Thesis

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Logistics Management

By

Dae Won Kim, BS
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CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

Statement of the Problem

An inventory problem exists whenever it is necessary to stock physical goods or commodities for the purpose of satisfying demand over a specified period of time. Almost every business must stock goods to ensure the smooth and efficient running of its operation. Decisions regarding how much and when to order are typical of every type of inventory problem. The answer to this type of problem depends on a large number of factors, such as the demand pattern for the commodity, circumstances governing its replenishment, various inventory costs, and the characteristics of the commodity such as whether it is flammable, poisonous, explosive, perishable, or deteriorating. Of these important factors, the notion of item deterioration has not been adequately addressed in the literature.

In general, almost all items deteriorate over some time period. Fortunately, for most items, the rate of deterioration is so slow that there is little need to consider the factor of deterioration when determining economic lot sizes. However, commodities such as blood, alcohol, gasoline, and certain foods are examples of perishable

products that deteriorate rapidly over time. Since these types of products deteriorate relatively quickly in inventory, the cost impact of their loss should be considered. Many researchers have developed various inventory models to reduce losses due to deteriorating inventories. However, there is no existing model which treats the demand price function, which is essential to the market in facilitating optimal price and production level deteriorations. The focus of this effort is on the modification of an inventory model to facilitate optimal price and production level determinations.

Literature Review

This review examines the current literature to show the flow of inventory model development for a deteriorating inventory system and provides a basis for modifying the current model.

Initial researchers considered optimal production decisions when developing the deteriorating inventory model. Later, other reseachers added optimal price decisions to the inventory problem. Recently, many efforts have analyzed mathematical models of inventory for items with a stochastic lifetime, and deteriorating life cycle in inventory.

When developing inventory models, initial researchers were concerned with optimal production decisions only.

Ghare and Schrader (5) described several models in which depletion over time was the result of the combined effect of demanded usage and decay; such as direct spoilage as in fruit, physical depletion as in highly volatile liquids, or deterioration as in radioactive substances, blood banks, and grain. They derived a revised economic ordering quantity (EOQ) model under conditions of constant demand and exponential decay. Emmons (4) developed a replenishment model for radioactive nuclide generators which also modeled exponential decay where a product which decayed at one rate was processed into a new product which decayed at a second rate. His model particularly applies to inventories of radioactive isotopes. Covert and Philip (3) developed the EOQ model for items with a variable rate deterioration by utilizing the two-parameter Weibull distribution, which is assumed to be useful for an item with a decreasing rate of deterioration only if the initial rate is either an extreme high or low. In addition, Philip (11) expanded Covert and Philip's model and developed a generalized EOQ model by assuming the three-parameter Weibull distribution to describe the time to deterioration of an item. Specifically, the three-parameter Weibull distribution can be used for items with any initial values for rate deterioration and also for items which start deterioration only after a certain period of time. Hence, a more general EOQ

model was developed using a three-parameter Weibull distribution.

Cohen (1) was also concerned with an inventory problem in which the product is perishable. In particular, the product is distinguished by a maximum usable lifetime. Impetus for the analysis came from attempts to apply existing perishable inventory theory to the practical problems associated with blood bank management. Specifically, he considered the effects on the inventory model of restricting order policy to the single critical number class, which is an ordering quantity thought to be optimal for an item. The objective function was expected cost per period. Accordingly, the steady state characteristics of the inventory process, influenced by the order restriction, were analyzed. He demonstrated the existence of an invariant measure for an inventory-related process, which provides information sufficient for cost minimization.

In the production lot size for the deteriorating inventory system, Misra (9) developed a more general and realistic deterministic model for items with either a constant or a variable rate of deterioration for a system with a finite production rate. He used a two-parameter Weibull rate such that the items in inventory start deteriorating the instant they are received into inventory. He showed the impact of a constant deterioration rate on the production lot size model. His results reduced the optimum

production lot size and also reduced their associated costs.

The consideration of price as an inventory decision variable has been undertaken by a number of authors. Whitin (16) showed the important relationship between the inventory control and price policy. Most of the inventory control systems consider the determination of economical lot sizes only with cost minimization aspects of the problem, and neglects the demand function which is a standard tool of economic theory. He described the effect of the demand function on inventory control levels and optimization with respect to both price and stocked level. In short, the analysis has linked price policy and inventory control policy together in various models and has determined a combined policy which yields the highest profits.

Kunreuder and J. F. Richard (7) described optimal pricing and inventory decisions for non-seasonal items. They investigated the relationship between the pricing and inventory decisions for a retailer who orders his goods from an outside distributor. Most retailers make pricing decisions at certain times of the year on the basis of the expected demand for their product. In order to do this, they have some idea of what their demand curve looks like over some range of prices, however narrow this range may be. The firm may, of course, revise its initial decision in the future if certain unanticipated changes in demand or costs

occur. The marketing department would want to meet the retailer's demand and maximize the profits under the assumption that the inventory-related costs were zero. The purchasing department would then specify an ordering policy based on price which minimized the inventory-related costs. In this sense, the decisions with respect to price and order size should be considered for optimal inventory control.

Kunreuder and L. Scharage (8) showed the joint pricing and inventory decisions for constant priced items by expanding the results of the study by Kunreuder and Richard (7) to a more interesting case. They developed an algorithm for determining the pricing and ordering decisions for a firm that produces one product for which there is a deterministic demand curve that differs from period to period. It is assumed that the firm wants to maintain the same price for the product throughout the season. There is a fixed cost associated with each order placed in addition to per unit ordering and storage costs for carrying inventory over time.

Early simplistic mathematical techniques did not provide a suitable method for manipulating complicated deteriorating functions in algorithms which addressed perishable items. However, the differential equation provides a means of handling these functions. Spiegel (13) showed how the differential equation could be used to

manipulate functions of deteriorating inventory containing time, quantity, and cost.

So far, we have considered the flow of the development of the deteriorating inventory models and the importance of the demand function which has been usually neglected by businessmen. Even though there are many kinds of deteriorating inventory models, most of those models have ignored the impact of the demand function in attaining an optimal price and inventory policy decision. Only Cohen (2) expanded his early model by considering Kunreuder and Scharage's joint pricing and inventory policy theory in the EOQ model for the decaying inventory system. In the production lot size model for the deteriorating inventory system, Misra (9) developed a more general and realistic model by using the two-parameter Weibull distribution. Misra's model, however, used a simple cost function which ignored the impact of the demand function (market controlled price or selling price). Therefore, the model itself lacked accuracy in determining optimal price and production lot sizes.

In summary, the initial research to develop deteriorating inventory models concerned only optimal production decisions, and the consideration of price as an inventory decision variable was later added to those inventory models. However, most of the deteriorating inventory models ignore the impact of the demand price function. Misra (9) developed a more general and realistic production

lot size model for a deteriorating inventory system. Misra's model was inaccurate because the model was developed by using a simple cost function which neglected the demand price function. Therefore, no model currently exists in the literature which can determine, with improved accuracy, the optimal price and production level of inventory with deteriorating characteristics.

Objective

The objective of this research is to determine if an existing inventory model may be modified such that price and production levels for a deteriorating inventory system can be optimized.

Scope and Limitations

This study, which considers current literature on the deteriorating inventory problem, contains the following limitations for developing the deteriorating inventory model. Generally, constant deterioration of an item was observed to follow the exponential distribution by Misra (9); therefore, the exponential distribution represents the distribution of the time to deteriorate constantly. The optimal production quantity is assumed to be under conditions of continuous review, deterministic demand of a constant rate and no shortage because stochastic demand and shortage make this problem much more complicated. The sensitivity to changes in perishability and product

price is considered. From these materials, a revised deteriorating inventory model for optimal price and production level will be developed. Current studies in selected text and reference books provide detailed material for development of mathematical calculations, and application of this deteriorating inventory model. These studies were identified in the literature review.

Research Questions

1. Can an inventory model be modified such that price and production level for a deteriorating inventory system will be optimized?

2. Does the modified model have an impact on price and deterioration?

3. Can optimal price and production levels be attained?

CHAPTER II

RESEARCH METHODOLOGY

Introduction

This methodology chapter will develop the procedure for answe·ing the research questions mentioned earlier, such as modifying the model to attain an optimal price and production level for a deteriorating inventory system, checking an impact on price and deterioration for the modified model, and attaining the optimal price and production levels. First, Misra's model as a general and realistic model will be presented to establish a foundation for understanding how his model treats the deteriorating production lot size. The modified model will be developed on the basis of this model and will be illustrated in Chapter III. Further, the faults of the Misra model will be indicated. The discussion of the mathematical development will describe how the concept of the price function can be added to the existing model and how the differential equations for the modified model may be solved. A numerical example will describe how to validate the modified model and also show the impact on price and deterioration for the modified model.

General Model

Misra's model of deteriorating inventory systems represents the most current evolution of these type models found in the literature. The purpose of presenting Misra's model is to describe it and to indicate its faults with respect to handling price functions. Misra (9) developed a general and realistic production lot size model for items with either a constant or a variable rate of deterioration. Misra used a two-parameter Weibull rate which can be applied to those items that may start deteriorating the instant they are received into inventory. For the more general case, a three-parameter Weibull rate would be used for already deteriorated items and also for those items which may start deteriorating sometime in the future. For mathematical simplicity, the two-parameter Weibull rate has been used in Misra's model. Most of the following is introduced directly from Misra's article (9); some explanations are added to help understand the equations, and the order of equations is also changed to help understanding. Of his two models, Misra's variable deterioration rate model is omitted because this paper concerns, specifically, the constant deterioration rate model.

Description of Variables

The variables used in Misra's model are as follows:

ϕ = production rate given in number of units/year

λ = demand rate given in number of units/year

Q = production lot size

I = the inventory level at time t

I_0 = maximum inventory level within a cycle

C = cost of a deteriorated unit

C_1 = inventory holding cost/unit/unit time

C_3 = setup cost/cycle

T = cycle time

T_1 = time required to produce Q units

T_2 = time during which there is no production in a cycle; i.e., $T_2 = T - T_1$

$D(t)$ = the deterioration rate, given by $\alpha\beta t^{\beta-1}$ where $\alpha,\beta,t > 0$. When $\beta = 1$, $D(t)$ becomes a constant which is the case of an exponential decay

K = total cost/unit time

Assumptions

Misra's model was developed using the following assumptions:

1. Demand is known and has a constant rate.
2. Shortages are not allowed.
3. Production rate governing supply is finite.
4. Units are available for satisfying demand immediately after their production.
5. A deteriorated unit is not repaired or replaced by a good unit.

6. The cost of a deteriorated unit is constant.

7. The units start deteriorating only when they are received into inventory. This assumption allows us to use a two-parameter Weibull rate as mentioned earlier.

8. The system is in steady state; i.e., the production rate is greater than the demand rate.

9. The production lot size, though unknown, is fixed; i.e., it will not vary from one cycle to another.

Mathematical Development

The initial and ending inventory level of the cycle is assumed to be zero. The cycle length is equal to Q'/λ where Q' is the number of good units from Q units which is the production lot size. The production will take place for a duration of T_1 time units; the time required to produce Q units. At the end of this period, enough units should be on hand to cover the demand and the losses due to deteriorated units which occur in time unit T_2, during which there is no production in the cycle.

Inventory Level (I). Let $D(t)$ represent the deterioration rate function for the item stocked. The change in the inventory level, dI during a very small interval of time dt, is a function of the deterioration rate $D(t)$, the demand rate λ, production rate ϕ and the remaining inventory. I denotes the inventory at time t.

Therefore, the infinitesimal change in the inventory level at time t_1 during production is:

$$-dI_1 = ID(t)dt + \lambda dt - \phi dt \text{ for } 0 \leq t_1 \leq T_1, \tag{1}$$

and the infinitesimal change in the inventory level at time t_2 during no production is:

$$-dI_2 = ID(t)dt + \lambda dt \qquad \text{for } T_1 \leq t_2 \leq T. \tag{2}$$

Equations (1) and (2) can be rewritten as

$$\frac{dI_1}{dt} + ID(t) = (\phi - \lambda), \qquad 0 \leq t_1 \leq T_1, \tag{3}$$

and

$$\frac{dI_2}{dt} + ID(t) = -\lambda, \qquad T_1 \leq t_2 \leq T. \tag{4}$$

The solutions of these differential equations are given in Spiegel (10). Therefore, the inventory level at time t_1 during production is given by:

$$I_1 = \frac{\int_0^{t_1} (\phi - \lambda) \exp(\int D(t)dt)dt + B_1}{\exp(\int_0^{t_1} D(t)dt)} \tag{5}$$

and inventory level at time t_2 during no production is given by:

$$I_2 = \frac{\int_{T_1}^{t_2} (-\lambda) \exp(\int D(t)dt)dt + B_2}{\exp(\int_{T_1}^{t_2} D(t)dt)} \tag{6}$$

The values of the constants of integration B_1, B_2 can be found by establishing and using the boundary conditions. That is, the lower boundary is established by the initial inventory such that $t_1 = 0$ and $I_1 = 0$ and the upper boundary is expressed as $t_2 = T_1$ and $I_2 = I_0$. Applying these boundary conditions yields $B_1 = 0$, $B_2 = I_0$. This gives

$$I_1 = \frac{\int_0^{t_1} (\not{c}-\lambda)\ \exp(\int D(t)dt)dt}{\exp(\int_0^{t_1} D(t)dt)} \tag{7}$$

and

$$I_2 = \frac{\int_{T_1}^{t_2} (-\lambda)\ \exp(\int D(t)dt)dt + I_0}{\exp(\int_{T_1}^{t_2} D(t)dt)} \tag{8}$$

In order to simplify the expressions of I_1, I_2 further, it is imperative that the deterioration rate function $D(t)$ be known.

If the deterioration rate is constant, the function $D(t)$ can be written as $D(t) = \alpha$. Substituting this value of $D(t)$ in equations (7) and (8) yields the inventory level at time t_1 and t_2:

$$I_1 = \frac{\int_0^{t_1} (\not{c}-\lambda)\ \exp(\alpha t)\ dt}{\exp(\alpha t_1)}$$

$$= \frac{(\not{c}-\lambda)}{\alpha}\ [1 - \exp(-\alpha t_1)], \tag{9}$$

$$I_2 = \frac{(-\lambda)\ \exp(\alpha t)dt + \int_0^{T_2} \lambda \exp(\alpha t)dt}{\exp(\alpha t_2)}$$

$$= \frac{\lambda}{\alpha} \cdot \left[\frac{\exp(\alpha T_2) - \exp(\alpha t_2)}{\exp(\alpha t_2)}\right]. \tag{10}$$

Now at $t_2 = T - T_1 = T_2$, $I_2 = 0$; hence the maximum inventory level at time t_2 is

$$I_0 = \int_0^{T_2} \lambda \exp(\alpha t)dt$$

$$= \frac{\lambda}{\alpha} [\exp(T_2) - 1] \tag{11}$$

Total Cost (K). Let C_1 represent holding cost/unit/unit time, C_3 represent setup cost/cycle, C represent the cost of a deteriorated unit and K represent total cost/unit time.

Holding Cost. The total holding cost/unit time can be written as follows:

$$C_1 \frac{1}{T_1 + T_2} \left[\int_0^{T_1} I_1 dt + \int_0^{T_2} I_2 dt\right],$$

after substituting I_1 and I_2 in equations (9) and (11), the total holding cost/unit time yields

$$C_1\left[\frac{1}{T_1+T_2}\int_0^{T_1}\frac{(\not{c}-\lambda)}{\alpha}\,[1-\exp(-\alpha t_1)]dt_1\right.$$

$$\left.+\int_0^{T_2}\frac{\lambda}{\alpha}\left(\frac{\exp(\alpha T_2-\exp(\alpha t_2)}{\exp(\alpha t_2)}\right)dt_2\right],$$

or, after simplification

$$\frac{C_1}{T_1+T_2}\left[\left(\frac{\not{c}-\lambda}{2}\right)T_1^2+\frac{\lambda T_2^2}{2}\right].$$

<u>Deteriorated Unit Cost</u>. The cost of deterioration/unit time (C) is

$$\frac{C((\not{c}-\lambda)T_1-I_0)}{T}+\frac{C(I_0-\lambda T_2)}{T}$$

or, after simplification

$$\frac{C\not{c}\,T_1}{T_1+T_2}-\lambda C\ .$$

<u>Setup Cost</u>. The setup cost/unit time is

$$\frac{C_3}{T_1+T_2}\ .$$

Summing all these three costs gives the total cost/unit time

$$K=\frac{C\not{c}\,T_1}{T_1+T_2}-\lambda C+\frac{C_1}{2}\cdot\frac{[(\not{c}-\lambda)T_1^2+\lambda T_2^2]}{T_1+T_2}+\frac{C_3}{T_1+T_2}\qquad(12)$$

As can be seen, the total cost contains three costs: the cost of holding inventory, the cost of a deteriorated unit, and the setup cost. Therefore, because Misra's model was developed by using a simple cost function which neglected the demand price function, the impact of price and deterioration on production lot size level cannot be determined.

Development of the Model

This section will describe how the concept of the price function can be added to the existing model and then show how the differential equations of the modified model may be solved.

Mathematical Development

This mathematical development section will describe how the price function can be added to the existing model and how the differential equation for the modified model in this type of inventory system may be solved. The first step is to develop the ideal concept for adding the price function to the current model. The concept comes from Thowsen (15). The elements of the concept are that the demand function is derived by the price function because price can control demand. However, the control of the demand by price is not absolutely true in all cases. But, for this, the demand price function is assumed to have a negative slope; i.e., when price is increased, the demand is

decreased. After deciding the concept for adding the price function, the next step is the process of mathematical development. Some variables and assumptions will be added or changed at this step during development. This process of mathematical development will require the solution of differential equations. It is usually hard to solve differential equations when the equation contains integrals.

There are many kinds of methods to attack this problem. The simplest method is the tedious and long method of expanding the exponential terms in a series form and then integrating term by term. However, since most series usually contain an infinite number of terms, generally, most researchers (1; 3; 5; 8; 9; 11; 16) have resolved to ignore terms with second and higher order differentials to solve this type inventory model because the effect of higher order terms is negligible. Ignoring the second and higher order differential equations will result in a simple, first order differential expression which can be differentiated and equated to zero. Thus, the approximate optimum value can be found. Covert and Philip (3) have applied the Correction Method of Newton (14:79-83) which uses the recursive formula in geometric terms for their EOQ model with a Weibull distribution. This method or bisection method (6; 10) can be used for a constant deterioration such as the case of exponential decay (9:496)

because the Weibull rate can be changed to a constant form by setting β equal to one.

From the above discussion, it is clear that obtaining the solution of differential equations for our model is not easy. Therefore, if the second and higher order differential equations are ignored, as most researchers have done, the problem can be considerably simplified to get an approximation of the optimum value. The computer package for solving differential equations and high order nondifferential equations can also be used for this study. Some of the most useful of these include DSL/90, MIMIC, BHSL, DIHYSYS, and S/360 CSMP (12:119).

Validating the Model

Numerical examples will be prepared to facilitate validating the modified model. The modified model will be validated by demonstrating how the production lot size of a deteriorating inventory system is affected by the introduction of the price function under the given deterioration rate. These values for the variables of the numerical example will be arbitrarily given by considering some specific deteriorating item such as a shelf life item in the Department of Defense, assuming there is an appropriate item which satisfies the given assumptions.

The concern of market entry for setting price in the event of facing inventory costs and a downward sloping

demand curve was considered by Kunreuder and Richard (7). The producer will influence the market by setting price to obtain a positive profit. The optimal price will be achieved at some finite price strictly greater than total production cost. The producer will adjust optimal price to remain profitable. Therefore, it is important to show the impact of the price and deterioration on a production lot size model and to obtain the optimal price and production level for attaining maximum profit under a given deterioration rate for a specific item.

This demonstration by numerical example will also show how the modified model will be used to answer the research questions to meet the research objective.

CHAPTER III

DEVELOPMENT OF THE MODEL

This chapter will describe the variables, assumptions and mathematical development of how the concept of the price function can be added to the existing model and then how the differential equations of the modified model may be solved.

Descriptions of Variables

The variables used in this paper are as follows:

ϕ = production rate in number of units/day

*$d(p)$ = demand rate, when the price is p, in number of units/day

*C = unit production cost

C_1 = holding cost/unit/unit time

C_2 = setup cost per cycle

I_t = the inventory level at time t

I_0 = maximum inventory level within a cycle

T = cycle time

T_1 = production time per cycle

T_2 = time during which there is no production in a cycle; i.e., $T_2 = T - T_1$

α = the deterioration rate, α is a constant which is the case of an exponential decay

Q = production lot size

TC = total cost/unit time

$*T_1*$ = optimal value of T_1

$*T_2*$ = optimal value of T_2

$*T_{1c}*$ = optimal value of T_1 for conventional production lot size model

$*T_{2c}*$ = optimal value of T_2 for conventional production lot size model

Q = optimal production lot size

Some of the above variables are already used in Misra's model. This paper will also use those variables because they are general variables in this type of inventory model. The symbol * before the variables means the variable is an added one or a revised concept of the variables used in the Misra model.

Assumptions

The model will be developed using the following assumptions:

*1. Demand rate is known and constant. When the price is increasing, the demand rate is decreased; i.e., demand function d(p) = a - bp, where a, b are zero or positive coefficients (15:461-476).

*2. Production rate governing supply is finite and constant.

3. Shortages are not allowed.

4. Units are available for satisfying demand after their production.

5. A deteriorated unit is not repaired or replaced by a good unit.

*6. The unit production cost, C, is also considered to account for the deterioration cost, and all the cost coefficients are constant.

7. The system is in steady state; i.e., the production rate is greater than the demand rate.

8. Deteriorating rate follows exponential distribution with parameter α.

9. The units are deteriorating only when they are received into inventory.

10. The production lot size, though unknown, is fixed; i.e., it will not vary from cycle to cycle.

Some of the above assumptions are already used in Misra's model. This paper will also use those assumptions because they are general assumptions in this type model. The symbol * before the number of the assumption means the assumption is an added one or a revised concept of the assumptions used in Misra's model.

Mathematical Development

Figure 1 shows an inventory cycle for a finite production rate. The inventory level at the beginning and end of the cycle is zero. During time interval $(0, T_1)$ the inventory level increases due to production and decreases after production stops at time T_1. Let α represent the

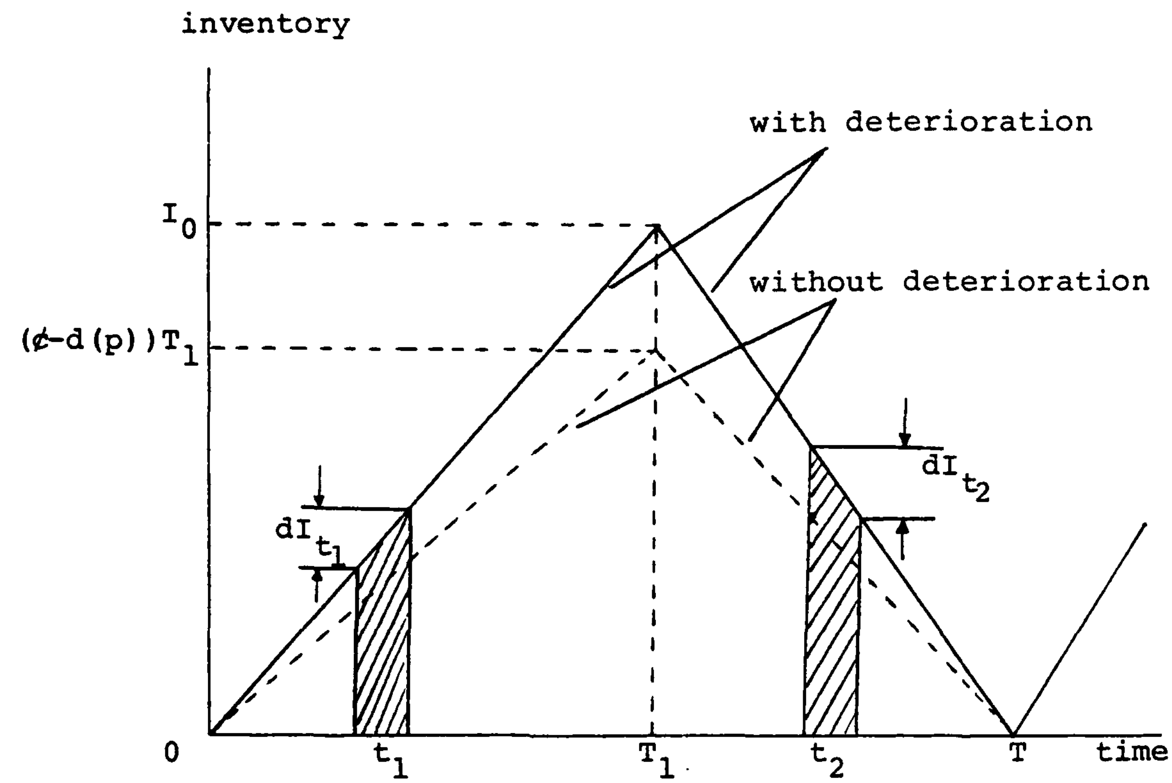

Fig. 1. A Finite Production Lot Size Model with Deterioration of Inventory

instantaneous deterioration rate function for the items stocked.

The change in the inventory level dI_t, during the infinitesimal time dt, is a function of the deterioration, the demand rate d(p), the production rate ϕ and the remaining inventory.

Thus

$$-dI_t = I_t \alpha dt + d(p)dt - \phi dt \quad \text{for } 0 \le t \le T_1 \tag{13}$$

and

$$-dI_t = I_t \alpha dt + d(p)dt \quad \text{for } T_1 < t \le T \tag{14}$$

Equations (13) and 14) can be rewritten as

$$\frac{dI_t}{dt}+\alpha I_t = \cent - d(p), \quad 0 \leq t \leq T_1 \tag{15}$$

and

$$\frac{dI_t}{dt}+\alpha I_t = -d(p), \qquad T_1 < t \leq T \tag{16}$$

To solve this first-order linear differential equation, multiply by an integral factor, both sides of equation (15).

$$\exp\left(\int_0^{t_1} \alpha dt\right) \frac{dI_t}{dt} + \alpha \exp\left(\int_0^{t_1} \alpha dt\right) I_t = (\cent - d(p)) \exp\left(\int_0^{t_1} \alpha dt\right) \tag{15-1}$$

such that

$$\frac{d}{dt}\left[\exp\left(\int_0^{t_1} \alpha dt\right) \cdot I_t\right] = (\cent - d(p)) \exp\left(\int_0^{t_1} \alpha dt\right) \tag{15-2}$$

integrate both sides and solve for I_t. Thus

$$I_t = \frac{(\cent - d(p)) \exp\left(\int \alpha dt\right) dt, + A_1}{\exp\left(\int_0^{t} \alpha dt\right)}, \quad 0 \leq t \leq T_1 \tag{17}$$

and, in a similar fashion from (16)

$$I_t = \frac{(-d(p)) \exp\left(\int \alpha dt\right) dt + A_2}{\exp\left(\int_{T_1}^{t} \alpha dt\right)}, \quad T_1 < t \leq T \tag{18}$$

(A_1 and A_2 are integral constants.)

Using the boundary conditions that at $t = 0$, $I_t = 0$ and at $t = T_1$, $I_t = I_0$, $A_1 = 0$ and $A_2 = I_0$, let $t_1 = t$ for $0 \leq t \leq T_1$

and

$$t_2 = t - T_1 \quad \text{for } T_1 < t \leq T$$

then

$$I_{t_1} = \frac{\int_0^{t_1} (\cancel{c} - d(p))\exp(\alpha t)dt}{\exp(\alpha t_1)}$$

$$= \frac{\cancel{c} - d(p)}{\alpha}[1 - \exp(-\alpha t_1)] \tag{19}$$

$$I_{t_2} = \frac{\int_0^{t_2} (-d(p))\exp(\alpha t)dt + I_0}{\exp(\alpha t_2)}$$

$$= \frac{-\frac{d(p)}{\alpha}[\exp(\alpha t_2) - 1] + I_0}{\exp(\alpha t_2)} \tag{20}$$

Since $t_1 = T_1$, $I_{t_1} = I_0$ and at $t_2 = T_2$, $I_{t_2} = 0$, it follows tha

$$I_0 = \frac{\cancel{c} - d(p)}{\alpha}[1 - \exp(-\alpha T_1)] = \frac{d(p)}{\alpha}[\exp(\alpha T_2) - 1] \tag{21}$$

and, from (20) and (21)

$$I_{t_2} = \frac{d(p)[\exp(\alpha T_2) - \exp(\alpha t_2)]}{\alpha \exp(\alpha t_2)} \tag{22}$$

from (21), it follows that

$$T_2 = \frac{1}{\alpha} \ln \left[\frac{\cancel{c}}{d(p)} - \frac{\cancel{c} - d(p)}{d(p)} \exp(-\alpha T_1)\right], \tag{23}$$

from (22)

$$\int_0^{T_2} I_{t_2} dt = \frac{d(p)}{\alpha} \left[-\frac{1}{\alpha} - T_2 + \frac{1}{\alpha} \exp(\alpha T_2)\right] \tag{24}$$

and from (19)

$$\int_0^{T_1} I_{t_1} dt = \frac{\cancel{c} - d(p)}{\alpha} \left[T_1 + \frac{1}{\alpha} \exp(-\alpha T_1) - \frac{1}{\alpha}\right]. \tag{25}$$

Equations (24) and (25) can be expressed in terms of T_1 using equation (23).

$$\int_0^{T_1} I_{t_1} dt + \int_0^{T_2} I_{t_2} dt = \frac{\cancel{c} - d(p)}{\alpha} T_1 - \frac{d(p)}{\alpha^2} \ln\left[\frac{\cancel{c}}{d(p)} - \frac{\cancel{c} - d(p)}{d(p)} \exp(-\alpha T_1)\right] \tag{26}$$

Thus, the total inventory cost/unit time becomes

$$TC(T_1, p) = \frac{C_2 + C\cancel{c}T_1 + C_1 \frac{\cancel{c} - d(p)}{\alpha} T_1 - \frac{C_1 d(p)}{\alpha^2} \ln\left[\frac{\cancel{c}}{d(p)} - \frac{\cancel{c} - d(p)}{d(p)} \exp(-\alpha T_1)\right]}{T_1 + \frac{1}{\alpha} \ln\left[\frac{\cancel{c}}{d(p)} - \frac{\cancel{c} - d(p)}{d(p)} \exp(-\alpha T_1)\right]} - d(p)c \tag{27}$$

To derive an approximate solution, those terms of degree higher than or equal to 2 in α are neglected in Taylor's expansion of the function (27) and assuming $\alpha < 1$; this reduces to

$$TC(T_1,p) = \frac{C_2 + C\not{c}T_1 + \frac{C_1}{2}\frac{\not{c}(\not{c}-d(p))}{d(p)}T_1^2}{\frac{\not{c}}{d(p)}T_1 - \frac{\alpha}{2}\left(\frac{\not{c}-d(p)}{d(p)}\right)\frac{\not{c}}{d(p)}T_1^2}$$

$$+ \frac{-\frac{\alpha\not{c}C_1}{3}\left(\frac{\not{c}-d(p)}{d(p)}\right)^2 T_1^3 - \frac{C_1\alpha}{6}\left(\frac{\not{c}-d(p)}{d(p)}\right)\not{c}T_1^3}{\frac{\not{c}}{d(p)}T_1 - \frac{\alpha}{2}\left(\frac{\not{c}-d(p)}{d(p)}\right)\frac{\not{c}}{d(p)}T_1^2} - d(p)C \qquad (28)$$

To optimize T, it is necessary to differentiate equation (28) with respect to T and then set the equation equal to zero,

that is,

$$\frac{\partial TC(T_1,p)}{\partial T_1} = 0,$$

Thus

$$-\frac{1}{3}C_1\alpha(2\not{c}-d(p))\frac{\not{c}}{d(p)}T_1^3 + \frac{1}{2}\not{c}(C\alpha+C_1)T_1^2 + C_2\alpha T_1 - \frac{C_2 d(p)}{\not{c}-d(p)} = 0 \qquad (29)$$

Equation (29) can be solved by computer using the bisection method (6; 10). See Appendices A and B for the computer programs to get optimal T_1* and T_2* values. But, here, for the purpose of developing the theory, letter style roots by approximation are needed to derive these values.

Therefore, assuming $\alpha T_1 < 1$, the above equation can be solved for the optimal T_1*.

This gives

$$T_1^* = \sqrt{\frac{2C_2 d(p)}{(\not{c}-d(p))\not{c}\ (C\alpha+C_1)}} \tag{30}$$

Recalling that T_1^* for items without deterioration $(\alpha \to 0)$ is

$$T_{1c}^* = \sqrt{\frac{2C_2 d(p)}{(\not{c}-d(p))\ \not{c}\ C_1}} \tag{31}$$

Equation (30) can be rewritten as

$$T_1^* = \sqrt{\frac{1}{\frac{C}{C_1}\alpha + 1}} \cdot T_{1c}^* \tag{32}$$

The same approximate procedures are applied to the equation which is expressed in terms of T_2. By setting

$\dfrac{\partial TC(T_2, p)}{\partial T_2} = 0$, then

$$\frac{C_1\not{c}}{6}\left(\frac{d(p)}{\not{c}-d(p)}\right)\alpha T_2^3 - \frac{\not{c}d(p)}{2}\left(\frac{C\alpha+C_1}{\not{c}-d(p)}\right)T_2^2 + C_2\alpha\left(\frac{d(p)}{\not{c}-d(p)}\right)T_2 + C = 0$$

This gives

$$T_2^* = \sqrt{\frac{2C_2(\not{c}-d(p))}{\not{c}d(p)\ (C\alpha+C_1)}} = \sqrt{\frac{1}{\frac{C}{C_1}\alpha + 1}} \cdot T_{2c}^* \tag{33}$$

where $T_{2c}^* = \sqrt{2C_2(\not{c}-d(p))/C_1\not{c}d(p)}$

Thus the cycle time T^* is the sum $T_1^* + T_2^*$ and the optimal production lot size is

$$Q^* = \not{c}T_1^* = \sqrt{\left(\frac{1}{\frac{C}{C_1}\alpha + 1}\right) \cdot \frac{2C_2 d(p)\not{c}}{(\not{c} - d(p))C_1}} \tag{34}$$

From equation (30), for fixed selling price p, the optimal production time decreases as decay rate α increases. Though demand d(p) has been assumed to decrease with increasing p, the effect of price on production cycle time is not known.

It is necessary to determine what impact deterioration and price variation may have on the optimal production decision. For comparative purposes the optimal production rate is examined.

From (19) and (30), by the Taylor approximate expansion,

$$I_{T_1^*}/T_1^* = \frac{\frac{\not{c} - d(p)}{\alpha}\,[1 - \exp(-\alpha T_1^*)]}{T_1^*}$$

$$= (\not{c} - d(p)) \cdot \left(1 - \frac{\alpha T_1^*}{2}\right)$$

where the first factor corresponds to the difference between the production rate and demand rate and second factor approximates the rate at which items deteriorate.

The sensitivity of the optimal production rate to changes in deterioration is determined by,

$$\frac{\partial}{\partial \alpha}\left(I_{T_1^*}/T_1^*\right) = [\cancel{c}-d(p)]\,\frac{-T_1^*}{2} - \left[\frac{\cancel{c}-d(p)}{2}\right]\alpha\frac{\partial T_1^*}{\partial\alpha}$$

$$= -\left[\sqrt{\frac{C_2 d(p)\,(\cancel{c}-d(p))}{2\cancel{c}(C\alpha + C_1)}}\cdot\frac{(C\alpha + 2C_1)}{(2C\alpha+2C_1)}\right] \leq 0 \qquad (36)$$

similarly, production rate is responsive to a price change as,

$$\frac{\partial}{\partial p}\left(I_{T_1^*}/T_1^*\right) = d'(p)\left[\frac{\alpha T_1^*}{4} - 1 - (\cancel{c}-d(p))\,\frac{T_1^*}{4d(p)}\right] \geq 0. \qquad (37)$$

It is seen that the optimal production rate decreases with an increase in deterioration rate α and also increases with increasing price, when assuming price to be an external (market - controlled) parameter.

It is important to verify that the response of optimal cycle time and production rate to changes in both price p and deterioration rate α is consistent with the results derived from the approximate cost function. An example problem was considered by solving (30) for T_1^*. The corresponding optimal production rate $I_{T_1^*}/T_1^*$ was then computed. This computer program is added in Appendix C. The results of the computation and the associated values of the cost parameters and values of p and α are illustrated in Table 1.

TABLE 1

OPTIMAL PRODUCTION TIME AND PRODUCTION RATE

p	Alpha (α)					
	0.01	0.03	0.05	0.07	0.09	0.12
5.0	4.0050 26.949	3.9290 25.879	3.8569 24.848	3.7887 23.853	3.7239 22.892	3.6327 21.506
10.0	3.6155 29.458	3.5466 28.404	3.4816 27.389	3.4199 26.409	3.3615 25.462	3.2791 24.098
15.0	3.2493 31.972	3.1874 30.946	3.1189 29.958	3.0735 29.004	3.0210 28.082	2.9470 26.753
20.0	2.8989 34.493	2.8436 33.507	2.7915 32.557	2.7420 31.641	2.6952 30.755	2.6292 29.479
25.0	2.5565 37.021	2.5078 36.089	2.4618 35.192	2.4183 34.326	2.3769 33.489	2.3187 32.283
30.0	2.2140 39.557	2.1719 38.697	2.1320 37.868	2.0943 37.068	2.0585 36.295	2.0080 35.181

Note: For example with C_2= \$250, C = \$1/unit and C_1= 0.5/unit/day and production rate ¢ = 50 unit/day and demand rate function is downward slope, d(p) = 25 - 0.5p (unit/day).

The expected reactions; i.e.,

$$\frac{\partial T_1^*}{\partial \alpha} \leq 0, \qquad \frac{\partial T_1^*}{\partial p} \leq 0$$

$$\frac{\partial}{\partial \alpha}\left(\frac{I_{T_1^*}}{T_1^*}\right) \leq 0 \quad \text{and} \quad \frac{\partial}{\partial p}\left(\frac{I_{T_1^*}}{T_1^*}\right) \geq 0$$

were observed.

From equation (28),

$$T = \frac{\not{c}}{d(p)} T_1 - \frac{\alpha}{2}\left(\frac{\not{c}-d(p)}{d(p)}\right)\frac{\not{c}}{d(p)} \; T_1^2 \tag{38}$$

In order to express the total cost function by total cycle time (T), let $T_1/T = n$ be the fraction of the cycle in which there is production time. From equation (28), cost/unit time can be expressed as a function of (T,n,p) as follows:

$$TC(T,n,p) = \frac{C_2}{T} + C\not{c}n + \frac{C_1}{2}\frac{\not{c}(\not{c}-d(p))}{d(p)} n^2 T - \frac{C_1\alpha}{3}\left[\frac{\not{c}-d(p)}{d(p)}\right]^2 \not{c}n^3 \; T^2$$

$$- \frac{C_1\alpha}{6}\left(\frac{\not{c}-d(p)}{d(p)}\right)\not{c}n^3 T^2 - d(p)C \tag{39}$$

for fixed price p, TC(T,n,p) must be minimized with respect to T. That is,

$$\frac{\partial TC(T,n,p)}{\partial T} = -\frac{C_2}{T^2} + \frac{C_1}{2}\frac{\not{c}(\not{c}-d(p))}{d(p)} n^2 - \frac{2C_1\alpha}{3}\left[\frac{\not{c}-d(p)}{d(p)}\right]^2 \not{c}n^3 T$$

$$- \frac{2C_1\alpha}{6}\left(\frac{\not{c}-d(p)}{d(p)}\right)\not{c}n^3 T$$

this equation can be solved by using bisection method (6; 10) by computer; but, here, for the theory, the letter style expression of roots by approximation is again needed to compute an optimal price. Therefore, assuming $\alpha T < 1$ yields

$$-\frac{C_2}{T^2} + \frac{C_1}{2}\left(\frac{\not{c}(\not{c}-d(p))}{d(p)}\right) n^2 = 0$$

thus,

$$T = \sqrt{\frac{2C_2 d(p)}{\not{c}(\not{c}-d(p))C_1}} \cdot \frac{1}{n} \tag{40}$$

and from equation (38),

$$n = \frac{T_1}{T} = \frac{1}{\frac{\not{c}}{d(p)} - \frac{\alpha}{2}\left(\frac{\not{c}-d(p)}{d(p)}\right)\frac{\not{c}}{d(p)} T_1} \tag{41}$$

In order to consider the optimal price decision, it is defined that the profit function is presented as a function of cycle time and price,

$$\pi(T,n,p) = pd(p) - TC(T,n,p)$$

The price and production level problem is equivalent to

$$\text{maximize} \quad \pi(T,n,p)$$
$$\text{for } p \geq 0$$

using (39) and differentiating with respect to p to get optimal price,

$$\frac{\partial\pi(T,n,p)}{\partial p} = d(p) + pd'(p) - \frac{\partial TC(T,n,p)}{\partial p}$$

$$= d(p) + d'(p)\left[p + \frac{C_1 n^2 T \phi^2}{2d(p)^2} - \frac{2C_1\alpha\phi^2 n^3 T^2}{3}\left(\frac{\phi - d(p)}{d(p)}\right)\right.$$

$$\left. - \frac{C_1\alpha\phi^2 n^3 T^2}{6d(p)} + C\right] = 0$$

yields

$$p^* = -\frac{d(p)}{d'(p)} - \frac{CnT\phi}{2d(p)} + \frac{2C_1\alpha\phi^2 n^3 T^2}{3}\left(\frac{\phi - d(p)}{d(p)^3}\right)$$

$$+ \frac{C_1\alpha\phi^2 n^3 T^2}{6d(p)^2} - C \tag{42}$$

From equations (40) and (41), equation (42) can be rewritten as

$$p^* = \frac{1}{\phi - \alpha\sqrt{\dfrac{C_2\phi(\phi-d(p))}{2d(p)\,(C\alpha+C_1)}}}\left[-\frac{d(p)}{d'(p)}\left(\phi - \alpha\sqrt{\frac{C_2\phi(\phi-d(p))}{2(C\alpha+C_1)d(p)}}\right)\right.$$

$$\left. - \phi\alpha\sqrt{\frac{C_1\phi C_2}{2(\phi-d(p))d(p)}} + \frac{4C_2\alpha\phi}{3d(p)} + \frac{C_2\phi\alpha}{3(\phi-d(p))}\right] - C \tag{43}$$

In order to determine this p* value, the computer program using the bisection method (6; 10) was used and is added in Appendix D.

CHAPTER IV

NUMERICAL EXAMPLES

Numerical examples for the model are illustrated to both validate the model and show the impacts of price and deterioration.

The values of various variables are arbitrarily given and are as follows:

production cost, $C = \$1/\text{unit}$,

holding cost, $C_1 = \$0.5/\text{unit/day}$,

setup cost, $C_2 = \$250/\text{order}$,

production rate, $\not{c} = 50$ unit/day,

demand rate, $d(p) = 25 - 0.5\,p$ unit/day.

From equation (43), if it is assumed that d(p) belongs to the class of functions satisfying the following conditions

(i) $d'(p) \leq 0$; i.e., demand function is negative slope

(ii) $\lim_{p\to\infty} d(p) = 0$

(iii) $\lim_{p\to\infty} pd(p) = 0$,

then $\lim_{p\to\infty} \pi(T,n,p) = 0$.

The profit function ($\pi(T^*,n,p^*)$) achieves its maximum at some possibly infinite value where the total revenue

function $(p*d(p)) \geq$ the total cost function $(TC(T^*,n,p^*))$ and is represented by $\pi(T^*,n,p^*) = p^*d(p^*) - TC(T^*,n,p^*)$. It is noted that maximum profit may be zero, in cases where p^* approaches infinity.

The variation of the optimal solution, (p^*,T_1^*), to changes in deterioration rate α was investigated numerically for various cost coefficient configurations. Analysis of the previously discussed example indicates that optimal price and optimal production time do not behave monotonically with respect to α. These computational results are illustrated in Table 2 with optimal production lot size and optimal production.

Management concerns of market entry for setting price in the event of facing inventory costs and a downward (negative) sloping demand curve was considered by Kunreuder and Richard (7). The production company will have a share in the market only in those cases where price is set to obtain a positive profit. Optimal price will be achieved at some finite price strictly greater than total cost. As the deterioration rate α increases, the producer must adjust the optimal price to remain profitable. The possibility of positive profit decreases with higher values of α. It is important to note that while the optimal price and optimal production decisions do not behave monotonically to increases in deterioration rate α, there is a marked stability in the value of the optimal price. For the

TABLE 2

VARIATION IN OPTIMAL SOLUTION WITH RESPECT TO DETERIORATION RATE ALPHA (α)

Alpha (α)	optimal price p^*	optimal production lot size Q^*	optimal production time T_1^*
0.01	23.329	133.53	2.6705
0.03	23.540	130.27	2.6055
0.05	23.795	127.05	2.5409
0.07	24.106	123.80	2.4759
0.09	24.492	136.36	2.7273
0.10	24.722	118.73	2.3745
0.12	25.286	115.05	2.3010
0.14	26.065	110.87	2.2174
0.16	41.160	60.606	1.2121
0.18	39.356	66.176	1.3235
0.20	37.500	71.429	1.4286
0.25	32.759	83.333	1.6667
0.30	28.049	93.750	1.8750
0.50	11.538	125.00	2.5000
0.60	5.0000	136.36	2.7273

example in Table 2 it is observed that for low values of deterioration rate α the optimal reaction to an increased deterioration rate is to increase price. In the range of higher values for α an optimal reaction to increased deterioration rate is to decrease price.

For a specific example with deterioration rate and optimal price established as follows, the optimal production lot size Q*, optimal production time T_1* can be derived by applying the developed algorithms.

deterioration rate $\quad = 0.03$

optimal price $\quad p = 23.540$

Solution:

$$Q^* = \not{c}T_1^* = \sqrt{\frac{2C_2 d(p)\not{c}}{(\not{c}-d(p))C_1}\left(\frac{1}{\frac{C}{C_1}\alpha+1}\right)}$$

$$= \sqrt{\frac{2 \times 250 \times (25 - 0.5 \times 23.540) \times 50}{(50 - 25 + 0.5 \times 23.540) \times 0.5}}$$

$$\cdot \sqrt{\frac{1}{\frac{1}{0.5} \times 0.03 + 1}} = 130.27 \text{ units}$$

$$T_1^* = \frac{Q^*}{\not{c}} = \frac{130.27}{50} = 2.605 \text{ days}$$

$$T_2^* = \sqrt{\frac{2C_2(\not{c} - d(p))}{\not{c}d(p)\,(C\alpha + C_1)}}$$

$$= \sqrt{\frac{2 \times 250 \times (50 - 25 + 0.5 \times 23.540)}{50 \times (25 - 0.5 \times 23.540)(1 \times 0.03 + 0.5)}}$$

$= 7.242$ days

Cycle time, $T = T_1^* + T_2^* = 2.605 + 7.242$

$= 9.847$ days,

Actual demand during $T = Td(p)$

$= 9.847\ (25 - 0.5 \times 23.540) = 130.27$ units

Total deteriorated units in a cycle time

$= 130.27 - 130.27 = 0$ unit.

Thus, by using optimal price and deterioration rate, the number of deteriorated units is reduced to a minimum.

The iterative computer programs to get an optimal price, optimal production lot size and optimal production time is shown in the appendices. Therefore, if the deterioration rate and demand function of a specific item is known, then the optimal price, optimal production lot size and optimal production time for the specific item can easily be determined.

CHAPTER V

CONCLUSION

Summary and Conclusion

In general, almost all items deteriorate over some time period. Especially, since some types of products such as blood, alcohol, gasoline and certain foods deteriorate relatively quickly in the inventory; the cost impact of their loss should be considered. Many researchers have developed various inventory models to reduce losses due to deteriorating inventories. There are two kinds of development flow of the deteriorating inventory models. When developing the model, the initial research to develop deteriorating inventory models concerned only optimal production decisions and the consideration of price as an inventory decision variable was later added to those inventory models. Even though there are many kinds of deteriorating inventory models, most of these models have ignored the impact of the demand price function where demand is determined by price. Misra developed a more general and realistic production lot size model for a deteriorating inventory system. Misra's model was also inaccurate because the model was developed by using a simple cost function which neglected the demand price function. Therefore, no

model currently existed in the literature which could determine, with improved accuracy, the optimal price and production lot size with deteriorating characteristics. Therefore, the more accurate production lot size model was needed to determine if an existing inventory model may be modified such that price and production levels for a deteriorating inventory system could be optimized.

In order to answer the research questions, an extension of Misra's model is made to include the situation in which the demand rate is expressed as the function of price. To avoid making the problem much more complicated, a revised deteriorating inventory model for the optimal price and production level was developed under the assumptions that there was no shortage and demand was constant. This modified model including the demand price function was examined such that price and production levels for a deteriorating inventory system were optimized by solving differential equations. The exact values for optimal production time (T_1*), optimal production lot size (Q*), optimal production rate (I_{T_1*}/T_1*), optimal nonproduction time (T_2*), and optimal price (p*) were solved by computer using the bisection method (6; 10). For the purpose of developing theory, the approximation values for those optimal solutions were taken by letter style expression.

In the numerical example, the modified model was validated by demonstrating the impact on price and

deterioration which had a correlation effect and by attaining the optimal price (p*) and optimal production level (Q*) under the given deterioration rate and demand price function. That is, if the deterioration rate and demand price function were determined by the market for a specific item, then the optimal price (p*), optimal production lot size (Q*) and optimal production time (T_1*) could easily be determined.

The results indicate that the tradeoff of revenue and loss due to deterioration rate and demand function allow for a more exact pattern of pricing and production decisions. If the interaction of price considerations on the deterioration rate and the demand function is not accounted for, less than optimal pricing and production decisions result in reduced available inventory at an increased cost.

This paper is a step toward analyzing the interaction effect of deterioration with optimal pricing and production decisions. If the result of this research can be extended to the real specific deteriorating items, a tremendous amount of money may be saved in the field of deteriorating inventory production systems and defense materials (radioactive missile warheads, volatile specific petroleum products and foods, etc.).

An extension of the deterministic inventory model could be to consider the situation in which the deterioration follows a three-parameter Weibull distribution. Another could be to consider the case of the stochastic demand.

APPENDICES

APPENDIX A

COMPUTER PROGRAM FOR OPTIMAL T_1^*

```
        PROGRAM KIMJOB
       EXTERNAL FF
       A=0.001
100    B=10.0
       PRINT*,'     ENTER ALPHA VALUE : '
       READ*,AL
       PRINT*,'     ENTER PRICE : '
       READ*,P
       IF(AL.GT.1.)     GO TO 200
       CALL BISECT(FF,A,B,1.E-6,IFLAG,AL,P)
       IF(IFLAG.GT.1)       GO TO 100
       XI=(A+B)/2.
       ERROR=ABS(A-B)/2.
       C3=50.
       DP=25.-.5*P
       Q=C3*XI
       PR=(C3-DP)*(1.-AL*XI/2.)
       PRINT 600,XI,ERROR
       PRINT 650,Q,PR
                           GO TO 100
600     FORMAT(10X,'THE ROOT = ',E10.5,6X,'PLUS/MINUS = ',E10.5)
650     FORMAT(13X,'Q = ',E10.5,6X,'PR = ',E10.5)
200    STOP
        END
       FUNCTION FF(X,AL,P)
       C=1.
       C1=.5
       C2=250.
       C3=50.
       DP=25.-.5*P
       FF=(-C1*AL*(2.*C3-DP)*C3/3./DP)*(X**3.)+(C3+(C*AL*C1))*
      C(X**2.)+C2*AL*X-C2*DP/(C3-DP)
       RETURN
       END
       SUBROUTINE BISECT(F,A,B,XTOL,IFLAG,AL,P)
       IFLAG=0
       N=-1
       FA=F(A,AL,P)
      CHECK FOR SIGN CHANGE
       IF(FA*F(B,AL,P).LE.0)      GO TO 5
       IFLAG=2
       PRINT 601,A,B
601    FORMAT(10X,' A = ',E10.5,6X,' B = ',E10.5)
       RETURN
5       ERROR=ABS(B-A)
6       ERROR=ERROR/2.
C     CHECK FOR SUFFICIENTLY SMALL INTERVAL
       IF(ERROR.LE.XTOL)    RETURN
       XM=(A+B)/2.
```

```
C     CHECK FOR UNREASONABLE ERROR REQUIREMENT
      IF(XM+ERROR.EQ.XM)         GO TO 20
      FM=F(XM,AL,P)
      N=N+1
      CHANGE TO NEW INTERVAL
      IF(FA*FM.LE.0.)      GO TO 9
      A=XM
      FA=FM
                           GO TO 6
9      B=XM
                           GO TO 6
20     IFLAG=1
      RETURN
      END
```

APPENDIX B

COMPUTER PROGRAM FOR OPTIMAL T_2*

```
       PROGRAM KIMJOB
      EXTERNAL FF
100   A=0.001
      B=10.0
      PRINT*,'      ENTER ALPHA VALUE : '
      READ*,AL
      PRINT*,'      ENTER PRICE : '
      READ*,P
      IF(AL.GT.1.)      GO TO 200
      CALL BISECT(FF,A,B,1.E-6,IFLAG,AL,P)
      IF(IFLAG.GT.1)       GO TO 100
      XI=(A+B)/2.
      ERROR=ABS(A-B)/2.
      PRINT 600,XI,ERROR
                           GO TO 100
600    FORMAT(10X,'THE ROOT = ',E10.5,6X,'PLUS/MINUS = ',E10.5)
200   STOP
       END
      FUNCTION FF(X,AL,P)
      C=1.
      C1=.5
      C2=250.
      C3=50.
      DP=25.-.5*P
      FF=(C1*C3*DP*AL/6./(C3-DP))*(X**3)-(DP*C3*(C*AL+C1)/2.
     C/(C3-DP))*(X**2)+(C2*AL*DP/(C3-DP))*X+C2
      RETURN
      END
      SUBROUTINE BISECT(F,A,B,XTOL,IFLAG,AL,P)
      IFLAG=0
      N=-1
      FA=F(A,AL,P)
C     CHECK FOR SIGN CHANGE
      IF(FA*F(B,AL,P).LE.0)       GO TO 5
      IFLAG=2
      PRINT 601,A,B
601   FORMAT(10X,' A = ',E10.5,6X,' B = ',E10.5)
      RETURN
5      ERROR=ABS(B-A)
6      ERROR=ERROR/2.
C     CHECK FOR SUFFICIENTLY SMALL INTERVAL
      IF(ERROR.LE.XTOL)    RETURN
      XM=(A+B)/2.
C     CHECK FOR UNREASONABLE ERROR REQUIREMENT
      IF(XM+ERROR.EQ.XM)          GO TO 20
      FM=F(XM,AL,P)
      N=N+1
C     CHANGE TO NEW INTERVAL
      IF(FA*FM.LE.0.)      TO TO 9
```

```
590=      A=XM
600=      FA=FM
610=                         GO TO 6
620=9      B=XM
630=                         TO TO 6
640        IFLAG=1
650=      RETURN
660=20    END
```

APPENDIX C

COMPUTER PROGRAM FOR OPTIMAL PRODUCTION TIME AND PRODUCTION RATE

```
       PROGRAM KIMJOB
      REAL AL,C,C1,C2,C3
      REAL P,DP,T1,PR
100   PRINT*,'     ENTER ALPHA VALUE : '
      READ*,AL
      PRINT*,'     ENTER PRICE : '
      READ*,P
      IF(AL.GT.1.)      GO TO 200
      C=1.
      C1=.5
      C2=250.
      C3=50.
      DP=25.-.5*P
      T1=SQRT((2.*C2*DP)/(C3=DP)/C3/(C*AL+C1))
      PR=(C3-DP)*(1.-AL*T1/2.)
      PRINT 650,T1,PR
                        GO TO 100
650    FORMAT(13X,'T1 = ',E10.5,6X,'PR = ',E10.5)
200   STOP
      END
```

APPENDIX D

COMPUTER PROGRAM FOR OPTIMAL PRICE AND OPTIMAL PRODUCTION LOT SIZE

```
      PROGRAM KIMJOB
      EXTERNAL FF
100   A=0.
      B=49.99
      PRINT*,'     ENTER ALPHA VALUE : '
      READ*,AL
      IF(AL.GT.1.)      GO TO 200
      CALL BISECT(FF,A,B,1.E-6,IFLAG,AL)
      IF(IFLAG.GT.1)       GO TO 100
      XI=(A+B)/2.
      ERROR=ABS(A-B)/2.
      C=1.
      C1=.5
      C2=250.
      C3=50.
      DX3=25.-.5*XI
      Q=SQRT(2.*C2*DX3*C3/C1/(C3-DX3)/(C*AL/C1+1.))
      T1=Q/C3
      T2=SQRT(2.*C2*(C3-DX3)/C3/DX3/(C*AL+C1))
      PR=(C3-DX3)*(1.-AL*T1/2.)
      PRINT 600,XI,ERROR
      PRINT 650,Q,T1,T2,PR
                         TO TO 100
600    FORMAT(10X,'THE PRICE = ',E10.5,6X,'PLUS/MINUS = ',E10.5)
650    FORMAT(13X,'Q = ',E10.5,3X,'T1 = ',E10.5,3X,
     C'T2 = ',E10.5,3X,'PR = ',E10.5)
200   STOP
       END
      FUNCTION FF(X,AL)
      C=1.
      C1=.5
      C2=250.
      C3=50.
      DX1=-.5
      DX=25.-.5*X
      FF=-DX/DX1-(1./(C3-AL*SQRT(C2*C3*(C3-DX)/2./DX/(C*AL*C1))))
     C*(C3*SQRT(C1*C2*C3/2./(C3-DX)/DX)-4.*AL*C2*C3/3./DX-C2*C3
     C*AL/3./(C3-DX))-C-X
      RETURN
      END
      SUBROUTINE BISECT(F,A,B,XTOL,IFLAG,AL)
      IFLAG=0
      N=-1
      FA=F(A,AL)
C    CHECK FOR SIGN CHANGE
      IF(FA*F(B,AL).LE.0)        GO TO 5
      IFLAG=2
      PRINT 601,A,B
601   FORMAT(10X,' A = ',E10.5,6X,' B = ',E10.5)
```

```
      RETURN
       ERROR=ABS(B-A)
       ERROR=ERROR/2.
      CHECK FOR SUFFICIENTLY SMALL INTERVAL
      IF(ERROR.LE.XTOL)    RETURN
      XM=(A+B)/2.
C     CHECK FOR UNREASONABLE ERROR REQUIREMENT
      IF(XM+ERROR.EQ.XM)        GO TO 20
      FM=F(XM,AL)
      N=N+1
C     CHANGE TO NEW INTERVAL
      IF(FA*FM.LE.0.)      GO TO 9
      A=XM
      FA=FM
                           GO TO 6
9      B=XM
                           GO TO 6
20     IFLAG=1
      RETURN
      END
```

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